Enrollment No: _______________________Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Differential and Integral Calculus

Subject Code: 4SC04DIC1 **Branch: B.Sc. (Mathematics)**

Semester: 4 **Date: 20/09/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: **(14)**

a) Find $grad\ f$, where $f = \log(x^2 + y^2)$. (02)

b) $Evaluate: \int_0^1 \int_0^2 (x+y)dy\, dx$ (02)

c) $curl(grad\ \phi) =$ _______. (01)

d) Is $\vec{v} = (6y+8)i + (-6x+9)j$ irrotational? (02)

e) $Evaluate: \int_0^1 \int_0^2 \int_0^1 xyz\, dz\, dy\, dx.$ (02)

f) True/ False: At cusp two branches have a common tangent. (01)

g) State Stoke's theorem (02)

h) Define: Curvature (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions **(14)**

a) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$, given that $\frac{\partial z}{\partial y} = -2\cos y$ when $x = 0$ and $z = 0$ when y is a multiple of π. (05)

b) Change the order of integration in $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx\, dy$ and hence evaluate it. (05)

c) Evaluate: $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\, dz\, dy\, dx$ (04)

Q-3 Attempt all questions **(14)**

a) Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ at the point (3,1,2) in the direction of the vector $yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$. (05)

b) Given $\vec{u} = xyz\,\hat{\imath} + (2x^2z - y^2x)\,\hat{\jmath} + xz^3\,\hat{k}$ and $v = xy + yz + z^2$ then find $\nabla \cdot \vec{u}$ and $\nabla \cdot v$ and $\nabla \times \vec{u}$. (05)

c) Prove that $\vec{f} = \dfrac{x\,\hat{\imath} + y\,\hat{\jmath}}{x^2 + y^2}$ is solenoidal. (04)

Q-4 Attempt all questions **(14)**

a) Evaluate $\oint_C \bar{F} d\bar{r}$, where $\bar{F} = y^2\hat{\imath} + xy\hat{\jmath}$ and C is a square with vertices $(1,1), (-1,1), (-1,-1)$ *and* $(1,-1)$. (07)

b) Verify Stoke's theorem for $\vec{F} = xy^2\,\hat{\imath} + y\,\hat{\jmath} + z^2x\,\hat{k}$ for the suface of a rectangular lamina bounded by $x = 0, y = 0, x = 1, y = 2, z = 0$. (07)

Q-5 Attempt all questions **(14)**

a) State and prove Green's theorem. (07)

b) Verify the divergence theorem $\bar{V} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and S is a sphere $x^2 + y^2 + z^2 = 1$. (07)

Q-6 Attempt all questions **(14)**

a) Evaluate $\iint_R xydy\,dx$ where R is the first quadrant of the circle $x^2 + y^2 = a^2$. (05)

b) Evaluate $\int_0^2 \int_{y^2}^4 \left(x^2 + y^2\right) dx\,dy$ by change the order of integration. (05)

c) Find the equation of tangent plane and normal line to the surface $xyz = 6$ at the point $(1,2,3)$. (04)

Q-7 Attempt all questions **(14)**

a) Find $\int_0^2 \int_0^{\sqrt{2x-x^2}} \left(x^2 + y^2\right) dy\,dx$ by changing into polar co-ordinates. (05)

b) Using Green's theorem, evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $y^2 = x$ and $y = x^2$. (05)

c) Find the radius of curvature at any point on the curve $y^2 = 4ax$. (04)

Q-8 Attempt all questions **(14)**

a) Find the equation of tangent plane and normal line at point (3,4,5) to the surface $x^2 + y^2 - 4z = 5$. (05)

b) Form the partial differential equation by eliminating the arbitrary function from $z = xy + f\left(x^2 + y^2\right)$ (05)

c) Solve: $\dfrac{y^2 z}{x}\dfrac{\partial z}{\partial x} + xz\dfrac{\partial z}{\partial y} = y^2$ (04)